*J. Fluid Mech.* (1977), *vol.* 80, *part* 2, *pp.* 392–399
*Printed in Great Britain*

# REVIEWS

**Dynamic Meteorology.** Edited by P. Morel. Reidel, 1973. 622 pp. Dfl. 115 (about £28 or $43).

This is a collection of advanced lectures by a distinguished line-up of theoretical meteorologists: N. A. Phillips, J. G. Charney, D. K. Lilly, A. S. Monin, P. Morel and P. Queney. The lectures were delivered at a summer school held at Lannion, France, in 1970 and were subsequently 'expanded with extensive bibliographic references and some new developments' before publication in 1973. The volume will probably remain a significant addition to the literature of geophysical fluid dynamics for some time yet. It contains basic material unavailable elsewhere, particularly the lecture notes developed at M.I.T. by Phillips on 'principles of large scale numerical weather prediction' (100 pp.) and by Charney on 'planetary fluid dynamics' (250 pp.), which together occupy more than half the book.

Phillips is well known for his outstanding contributions to the early development of numerical weather prediction methods in the fifties. His lecture notes display the lucidity and attention to detail for which he is also well known. They greatly extend, in matters of formulation and technical detail, his 1970 article in the *Annual Review of Fluid Mechanics*, but give less background discussion. The overlap is thus fairly small, and the detail given here will be useful both as a reference for the professional and as didactic material for the student.

The opening sections form the best exposition of the basic equations I have yet come across, with greater than usual attention to the nature of the approximations involved, including the customary way in which spherical co-ordinates are used. The equations are also given in Mercator and stereographic co-ordinates, and in the two most widely used kinds of pressure 'co-ordinate' for hydrostatic atmospheres. A spin-off from the equations in stereographic co-ordinates is a mathematical derivation of the beta-plane approximation having the unusual merit of successfully disentangling dynamical from 'metric' effects.

Phillips then goes on to discuss dynamical 'balance', quasi-geostrophic motion and the problem of initiating numerical forecasts. Here he follows much the same line as in the *Annual Review* article, emphasizing by a 'WKBJ' analysis the wavelike nature of the small, high-frequency departures from geostrophic balance which are usually exaggerated by errors in the initial conditions and which frustrated L. F. Richardson's pioneering attempt at numerical weather prediction. The model serves its didactic purpose well; my only quibble concerns an omission to tell the reader that it assumes (not unreasonably for its purpose) a perfectly reflecting upper boundary. Such a boundary condition, as indeed is carefully mentioned elsewhere in both articles, is unlikely to be realistic for the actual atmosphere, even though it may well be all too 'realistic' for many forecasting models in use today.

There is finally a useful introduction to the numerical methods themselves, beginning with some basic theory on stability and convergence. It is not possible to fit onto the surface of a sphere a completely regular grid of more than 20 points – the vertices of a dodecahedron; Phillips' critical comparison of the compromises that have been tried is particularly useful. He also draws attention to the promise shown by spectral or Galerkin-type methods as an alternative answer to the problem. Since 1973 these spectral methods, implemented along lines suggested by S. Orszag, have been extensively tested for the sphere, notably by the U.K. Universities' Atmospheric Modelling Group at Reading, and have been found to be highly competitive with grid-point methods. Which will win in the long run is still an open question.

Charney is perhaps the most eloquent modern advocate of the classical idea that linear instability theories, for judiciously chosen basic flows, hold key insight into the behaviour of the atmosphere and its computer analogues. Two such theories, that of 'baroclinic instability' and that of 'conditional instability of the second kind' or 'CISK', in both of which Charney has been the principal pioneer or co-pioneer, are now widely believed to be valid epitomizations of energy conversion processes playing major roles in the large-scale circulations of the mid-latitude and tropical zones respectively. CISK is also thought to be the basic mechanism for the growth and maintenance of that smaller-scale but awesome tropical phenomenon, the hurricane. Other instability theories are important in thought-experiments: 'what would the atmosphere do *if*...'. So it is not surprising to find that instability theories are one of Charney's main themes; they are however only one facet of his wide-ranging and often successful search for penetrating idealizations of atmospheric behaviour. Other topics treated include simple viscous-fluid models of axisymmetric, thermally driven circulations on a sphere, geostrophic adjustment and balance, a brief treatment of planetary waves in the stratosphere (a subject which has evolved considerably since), frontogenesis, geostrophic turbulence, and a chapter on the general circulation of the atmosphere which synthesizes a number of the earlier ideas.

The early lectures on geostrophic balance and adjustment are perhaps the most extensively developed, and in interesting contrast to Phillips', and later Morel's, shorter and more mathematical treatments of the same topic. Here there are several levels of idealization and analogy, down to the humble mass-on-a-spring, which, when forced at low enough frequencies, 'passes through a succession of states of balance' just like a typical large-scale, geostrophically balanced flow evolving in time, a phenomenon which Charney and his collaborator A. Eliassen were among the first to understand clearly. The essential role of the ageostrophic 'secondary circulation' in such evolving flows, of which baroclinic instability and CISK both provide examples, is studied first in the simple case of two-dimensional or axisymmetric flow, before generalizing to three dimensions. The classical stability theory for axisymmetric disturbances suggests that the 'spring', i.e. the tendency for balance, is apt to be strong when the Richardson number is large. The two-dimensional case is important in its own right as an aid to thinking about CISK, about the zonally averaged atmospheric circulation, and about other phenomena such as quasi-two-dimensional frontogenesis.

To keep the notes up to date, many additions have been made over the years. The theory of frontogenesis is elegantly presented in its most modern form, due to Hoskins and Bretherton, although I wish space had been found for one of their very realistic model pictures of a strong front and associated jet stream forming near the tropopause, which for the first time showed just how simple the essentially fluid-dynamical explanation of this striking atmospheric phenomenon is. Another highlight, and one of the boldest of all recent ideas, is Charney's own 'geostrophic turbulence' theory, which, despite many unanswered questions, still seems to give the only sensible clue as to why the ideas of *two*-dimensional turbulence theory seem to predict the energy spectrum of *three*-dimensional, moderately large-scale atmospheric motions. (It is only on still larger scales, such that the Rossby height exceeds the tropopause height, that atmospheric motions are really quasi-two-dimensional.) One cannot of course find room for everything; but I was surprised to find no mention in the section on planetary waves of the celebrated 'Charney–Drazin theorem' and underlying 'Eliassen–Palm relations'. These powerful results warned us as long ago as 1961 of the circumstances under which wave transport effects, such as the apparent poleward eddy flux of heat due to vertically propagating planetary waves, which appears prominently in many discussions of the observed large-scale disturbances, may be entirely ineffective in actually changing the mean state because of other, compensating effects due to the same disturbances. I also thought that W. Blumen's result on stability of quasi-geostrophic shear flows could have found a place in the chapter on baroclinic instability: it is the most powerful available theorem on sufficient conditions for stability, and is not hard to extend to cover finite-amplitude disturbances. (While on the subject of baroclinic instability again, I should perhaps draw attention in passing to a minor error in J. S. A. Green's well-known growth-rate and phase-speed curves, reproduced on page 271, in the perhaps forlorn hope of retarding its persistent yet understandable propagation through the literature. The minimum growth rate at finite wavenumber should not be exactly zero, and the critical level should remain within the flow, with no continuous transition to retrograde neutral modes. More complicated behaviour occurs for larger values of $\beta$, as shown by Garcia & Norscini in a little-known paper in *Tellus* (1970), and is no doubt due to the presence of an upper lid in the model. Such complications are a typical bugbear of the normal-mode approach, and are of course irrelevant to the physically important faster-growing modes. The computer time expended on searching for very slowly growing modes is often out of all proportion to their physical interest; perhaps it ought to be standard practice to leave out the corresponding bits of the eigenvalue graphs, unless their structure can be found easily by analytical means.)

The notes are not always correct on other, less trivial, points of detail; one feels that, as with many great pioneers, Charney's intuition often runs far ahead of what can be obtained by detailed reasoning. For instance, on page 299, it is not literally true that baroclinic instability 'requires the existence of a surface temperature gradient', but it may well be true (and intuition is the only available guide here as far as I know) that surface temperature gradients can promote the growth of disturbances to large amplitudes, which is what matters in the context in question. For such reasons, and also because of a certain sparsity of 'connective tissue', the notes demand a fair amount of maturity and background knowledge on the reader's part. Professor Charney at one time had hopes of working them into the form of a more complete and permanent monograph – in the meantime we are lucky to have them in their present form in case his continuing interest in probing the many remaining mysteries in dynamical meteorology, oceanography, and climatology keep him from completion of such a project.

The remainder of the book is taken up with the shorter contributions by Lilly, Monin, Morel and Queney. Lilly gives a useful and readable survey of his work on numerical simulation of two-dimensional turbulence, which resulted among other things in some of the earliest numerical tests of Kolmogorov-type similarity arguments. Models of the atmosphere are set up by introducing forcing, or artificial feedback simulating the effect of baroclinic instability, at wavenumbers of a particular magnitude $k = k_F$. A term representing surface drag leads to an appropriate modification of Kraichnan's two-sided energy spectrum ($k^{-\frac{5}{3}}$ for $k < k_F$ and $k^{-3}$ for $k > k_F$). The drag term dissipates energy and vorticity-squared over substantial ranges of $k$, and gives rise to a largest scale, which appears to be not much greater than $k_F^{-1}$ for terrestrial parameter values. There is also a brief introduction to E. N. Lorenz's important arguments on the 'predictability problem', according to which the rate of propagation of errors from unobserved, very-small-scale motions into larger scales is sensitive to whether or not the spectral slope is less steep than $k^{-3}$. However, the numerical experiments are 'a bit inconclusive' as a test of Lorenz' predictions; some results of C. E. Leith suggest that much greater resolution is needed.

Lilly's figures 9.2 and 9.5 emphasize the intermittent nature of the generation of small scales of motion, and assume a special interest in the light of modern ideas about intermittency due to Landau, Mandelbrot, Frisch and others. These modify the Kolmogorov arguments because they suggest that the spectra do not, after all, become asymptotically independent of $k_F$. (It is therefore, incidentally, a bit alarming to read on p. 366 that 'many atmospheric and oceanographic scientists in small scale turbulence research tend now to test the calibration of new instruments and techniques on how well they produce a $-\frac{5}{3}$ power law'!) Figures 9.2 and 9.5 compare two ways of thinking about the excitation of small scales, and it is a pity that they were not constructed for the same initial conditions.

The article by Monin, on 'boundary layers in planetary atmospheres', reviews the similarity theory of turbulent boundary layers in stratified rotating atmospheres, a subject on which the author is an acknowledged authority. It is an adaption of an article published in 1970 in the *Annual Review of Fluid Mechanics*, 'with some additions'. The additions consist mainly of an all-too-brief discussion of what can be said, by means of simple, dimensional arguments, about the character of planetary atmospheric motions driven by solar radiation. Lilly's modification of Kraichnan's two-sided spectrum can be regarded as a development in much the same spirit, and he mentions Monin's lectures on planetary atmospheres as having been a source of inspiration. Morel gives a lively discussion of the interesting technical problems posed by the need for continuous 'four-dimensional' assimilation of real-time data into numerical forecasts, from the viewpoint of one who has been closely concerned with the topic on a practical as well as a theoretical level. Finally Queney expounds his classical work on the linear, analytical theory of two-dimensional mountain waves with and without rotation.

According to the editor's preface, 'the book gives a nearly complete and searching presentation of the theoretical basis of dynamic meteorology'. I would say that 'searching' is more appropriate than 'complete', especially when one recalls the complementary and very penetrating set of insights in Lorenz's classic monograph on the general circulation of the atmosphere, published by the World Meteorological Organisation in 1967. But the present volume does contain a convenient collection of more or less closely interrelated material, some of it very fundamental, some of it seen from more than one viewpoint, and some of it not readily available elsewhere. As such it will be of great value to a researcher wanting to improve his or her background or prepare a graduate lecture course. Some parts of it are also suitable as recommended reading for graduate students, given appropriate guidance. The editing and printing seem to have been done 'on the cheap', which would have been especially laudable had the price thereby been brought within reach of a research student's pocket.

M. E. McIntyre

**Theoretical Rheology.** Edited by J. F. Hutton, J. R. A. Pearson and K. Walters. Applied Science Publishers, 1975. 377 pp. £12.00.

This book contains the papers presented at the Autumn Conference of the British Society of Rheology, held in Cambridge in September 1974. The material is divided into four sections entitled 'Converging and Diverging Flow' (5 papers), 'Thermomechanics' (4), 'Composites and Suspensions' (5), and 'Rheometry' (6). A concluding report summarizes the discussions on the papers.

Two earlier conferences on Theoretical Rheology were organized by the British Society of Rheology in 1961 and 1971, but the proceedings of those meetings did not appear in book form. The decision to put the third conference in the series on permanent record is justified in a preface on two main grounds: first that the meetings on theoretical rheology have developed into significant international gatherings at which important current ideas are presented and discussed, and second that the papers collectively provide a readable survey of many of the most exciting modern developments in the field. One would not wish to dispute the first of these arguments, but readers led by the sweeping title of the book to expect a conspectus of the major research topics of the day in theoretical rheology are likely to be disappointed. No more than a quarter of the contributions could reasonably be described as surveys, most of the others being specialist research papers, not all of them with a significant *theoretical* content. The report of discussions contains nothing of lasting value.

The title is misleading, then, and, for the most part, the contents lack the representative and distinctive qualities claimed for them. Viewed more modestly, however, as a collection of papers on problems of contemporary interest in rheology the book serves to illustrate a number of important trends and themes. Of particular note are the increasing currency among rheologists of the language and methods of modern continuum mechanics, an expanding interplay between structural and phenomenological approaches to non-Newtonian flow, and the continuing controversy surrounding fundamental thermodynamic questions for materials with memory.

The most substantial article in the book is a survey by R. I. Tanner of recent progress in the experimental rheology of viscoelastic liquids. The discussion is largely concerned with viscometric and nearly viscometric flows and with the choice of a properly invariant constitutive equation capable of providing a unified and reasonably accurate account of the results. These flows are weak in the sense that, for a variety of molecular models, the rates of structural distortion enforced are small. At the end of his article Tanner reviews the much more fragmentary state of affairs regarding strong flows and, looking forward to further developments in this area and to more systematic studies of temperature-dependent effects, records his impression that progress will be made using a compound of microstructural physics and continuum mechanics, not by either in an essentially undiluted form. In the meantime exponents of the continuum approach will doubtless continue to find in particular flows of rheological fluids abundant scope for the application of analytical and numerical techniques. Papers in the present volume by J. R. Black *et al.*, S. Zahorski, K. Strauss and M. C. Phillips indicate the range of current activity in this field, and a survey article by G. Astarita & M. M. Denn, inappropriately placed at the end of the final section, provides a good introduction to the salient kinematic differences between Newtonian and non-Newtonian flows. The microstructural approach to non-Newtonian flow is represented by two papers, a survey by E. J. Hinch of the mechanics of fluid suspensions and a concise account by J. G. Evans of some simple flows of an ideal suspension of rigid rods. Hinch's article outlines the calculation of the bulk stress for a variety of suspensions, mostly dilute, in a Newtonian solvent. Adequate guidance to the literature is given here, but on turning at the end of his survey to rheological implications of the preceding results Hinch makes statements which are unsupported either by calculations or references. The reader seeking further enlightenment will find it necessary to consult papers by Hinch & Leal published in this *Journal* since the date of the conference.

Of the four papers on thermomechanics the first two, by R. S. Rivlin and J. G. Oldroyd, express dissatisfaction with existing attitudes to the thermodynamics of materials with memory. Rivlin's insistence that entropy must be a derived and not a primitive concept leads him to reject the well-known theory inaugurated by Coleman in 1964 and to propose an approach, restricted here to conditions of uniform temperature, patterned on Carathéodory's treatment of classical thermostatics. Oldroyd's paper is tentative and descriptive, but a mathematical formulation of the ideas is promised. Again it is appropriate to note that significant progress has been made since 1974 and that several versions of the thermodynamics of materials with memory are now available which allow the entropy to be constructed. The remaining papers in this section examined aspects of the thermomechanics of polymeric materials within the framework of Coleman's theory. M. J. Crochet provides a digest of a series of papers, written in collaboration with P. M. Naghdi, in which thermorheologically simple behaviour, associated with the idea of an equivalence between temperature and time, is characterized in the context of large deformations and non-isothermal conditions. Finally G. Astarita & G. C. Sarti give a general formulation of the property, long established in the physics of elastomers, that the accumulation of elastic energy is predominantly due to changes in conformational entropy. Concomitant simplifications in the stress constitutive equation and the residual energy equation are also discussed.

Space permits only a brief mention of the remaining survey article, a lucid account by T. G. Rogers of the mechanics of incompressible solids which are also inextensible along a family of embedded directions. The principal application of this theory is to fibre-reinforced elastic materials and Rogers show how an extensive class of solutions can be constructed by relatively simple means. Problems involving cracks are used to illustrate the methods and the paper concludes with a new solution for the finite opening of a linear crack.

There can be no doubting, on this evidence, that the meetings on theoretical rheology fulfil a useful purpose, but, to this reviewer at least, the advantages of publishing the proceedings in book form are much less clear.

P. CHADWICK

---

## Dynamics of Relaxing Gases. By J. F. CLARKE and M. MCCHESNEY. Butterworths, 1976. 576 pp. £25.00.

There have been many advances in real-gas dynamics since the appearance of Clarke & McChesney's book on *The Dynamics of Real Gases* in 1964. The new edition would therefore be expected to differ substantially from the first edition. In fact, the authors have gone further and completely rewritten and rearranged most of the text, so that we are really faced with a new book rather than a revision. As in the earlier book, the subject is limited by leaving out ionization and radiation effects, combustion and phase changes. Even within these limitations the selection of material reflects the authors' interests, as does the order in which it is presented. The reader would do well to study the preface very carefully before he embarks on the main text to get some idea of what to expect, as the chapter headings are not particularly illuminating.

In the first chapter the conservation equations are established on a molecular basis, and the laws of non-equilibrium thermodynamics are formulated from kinetic theory. Then a general model relaxing gas and its thermodynamics are introduced as a basis for the second chapter, which deals with certain aspects of the fluid dynamics of the relaxing gas, leaning heavily on the linearized approach, but including some of the nonlinear treatments with a discussion of Burgers's equation and the concept of bulk viscosity. A complete coverage has not been attempted, but even so, the treatment does at times, as elsewhere in the book, degenerate into a rather hurried listing of brief statements on the contents of published references. The chapter concludes with sections on energy transfer, but there is no discussion of boundary layers as such.

More than half of the book is made up of the last three chapters, a general treatise on collision processes, a chapter on vibrational relaxation of diatomic molecules, and the final and longest chapter on dissociation, recombination and reaction of diatomic molecules. These chapters constitute a thorough review of the present-day knowledge, or lack of knowledge, as the case may be, of intermolecular processes and their effects on macroscopic properties and rate processes. A brief introduction to quantum mechanics is followed by the wave mechanical treatment of collision processes leading to transition probabilities for the various types of energy exchange. The results are used on the vibrational relaxation of harmonic and anharmonic oscillators and there is also a brief discussion of rotational relaxation. The various theoretical approaches to dissociation, recombination and reaction of diatomic molecules are discussed and evaluated in considerable detail.

The book will be of considerable interest to the advanced theoretical worker, but it is my opinion that the graduate student, starting his study of the subject, would be well advised to start with a more easily digestible text, several of which are now fortunately available. The experimentalist will find no more than passing reference to experimental techniques and will have to look elsewhere for the actual numerical values of the properties of real gases. In this book references to experimental results are incidental rather than systematic, and one may perhaps express surprise that impurities get no more than a passing mention (as an experimental imperfection) in spite of their profound effect on the relaxation times of the commonest of all real gases, atmospheric air.

N. H. JOHANNESEN